3 (Sem–6/CBCS) STA HC 2

2022

STATISTICS

(Honours)

Paper : STA–HC–6026

(Multivariate Analysis and Nonparametric Analysis)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer ***any seven*** of the following questions as directed : 1×7=7

   (a) The moment generating function of bivariate normal distribution with parameters $\left(0, 0, \sigma_1^2, \sigma_2^2, \rho\right)$ is _____.

   (Fill in the blank)

(b) Let $\underset{\sim}{X} \sim N_p\left(\underset{\sim}{\mu}, \Sigma\right)$. Then the characteristic of $\underset{\sim}{X}$ is given by

(i) $e^{it\mu + \frac{1}{2}t'\Sigma t}$

(ii) $e^{it'\mu - \frac{1}{2}t'\Sigma t}$

(iii) $e^{it'\mu + \frac{1}{2}t'\Sigma t}$

(iv) None of the above

(Choose the correct option)

(c) Ordinary sign test considers the difference of observed values from the hypothetical median value in terms of:

(i) signs only

(ii) magnitudes only

(iii) sign and magnitude both

(iv) None of the above

(Choose the correct option)

(d) What is dispersion matrix in Multivariate data analysis?

(e) Let $(X, Y) \sim \text{BVND}\left(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho\right)$. Then state the conditional pdf of Y given $X = x$.

(f) What is run in non-parametric method?

(g) Define Multiple correlation coefficient.

(h) Let $\underset{\sim}{X} \sim N_3\left(\underset{\sim}{\mu}, \Sigma\right)$. Given that

$$\Sigma = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 0 \\ 3 & 0 & 4 \end{pmatrix}$$

Are X_2 and X_3 independent?

(i) The marginal distribution of a Bivariate normal distribution follows univariate normal distribution. *(State True or False)*

(j) The Kruskal-Wallis test is meant for:

(i) one way classification

(ii) two way classification

(iii) non classified data

(iv) None of the above

(Choose the correct option)

2. Answer **any four** of the following questions briefly: 2×4=8

(a) Define mean vector and dispersion matrix for multivariate data analysis.

(b) State the marginal pdfs of X and Y in case of $(X, Y) \sim \text{BVND}\left(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho\right)$.

(c) What assumptions are generally made for a non-parametric test?

(d) Let $\underset{\sim}{X} = (X_1 \;\; X_2 \;\; X_3)'$ have variance covariance matrix

$$\Sigma = \begin{pmatrix} 25 & -2 & 4 \\ -2 & 4 & 1 \\ 4 & 1 & 9 \end{pmatrix}$$

Find ρ_{12}.

(e) Define marginal distribution of $X_1, X_2, \cdots, X_k$ $(k < p)$ in a p-variate multivariate analysis. Also define the conditional distribution of $X_{k+1}, X_{k+2}, \cdots, X_p$ given $X_1, X_2, \cdots, X_k$.

(f) What indication can one get from the number of runs?

(g) Give a brief idea of Principal component analysis.

(h) The pdf of bivariate normal distribution is

$$f(x, y) = k \exp\left[-\frac{1}{2\left(1-\rho^2\right)}\left(x^2 - 2\rho xy + y^2\right)\right],$$
$$-\infty < (x, y) < \infty$$

Find the constant k.

3. Answer **any three** of the following questions: 5×3=15

(a) If $(X, Y) \sim \text{BVND}\left(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho\right)$, then show that X and Y are independent if and only if $\rho = 0$.

(b) Describe Kolmogorov-Smirnov one sample test stating its assumptions and hypotheses.

(c) Let $(X, Y) \sim \text{BVND}(0, 0, 1, 1, \rho)$. Then show that

$$Q = \frac{X^2 - 2\rho XY + Y^2}{\left(1-\rho^2\right)}$$

is distributed as chi-square with 2d.f.

(d) Let $\underset{\sim}{X} \sim N_p\left(\underset{\sim}{\mu}, \Sigma\right)$. Then find the distribution of $C\underset{\sim}{X}$ where C is a $p \times p$ non-singular matrix of constant elements.

(e) Write an explanatory note on test of randomness.

(f) With usual notations, prove that

$$r_{12\cdot 3} = \frac{r_{12} - r_{13}r_{23}}{\sqrt{\left(1-r_{13}^2\right)\left(1-r_{23}^2\right)}}$$

(g) Examine if Hotelling's T^2 is invariant under changes in the units of measurement.

(h) Describe one sample sign test for testing the null hypothesis that the population median is a given value.

4. Answer ***any three*** questions from the following: 10×3=30

(a) (i) State *any two* applications of multivariate analysis. 2

(ii) Let $(X, Y) \sim \text{BVND}\left(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho\right)$. Find the conditional distributions of $X/Y=y$ and $Y/X=x$. 8

(b) Derive the probability density function of p-variate normal distribution.

(c) (i) Describe the Wilcoxon Mann-Whitney U test. 5

(ii) Let $(X, Y) \sim \text{BVND}$ with parameters $\mu_x = 60$, $\mu_y = 75$, $\sigma_x = 5$, $\sigma_y = 12$ and $\rho = 0{\cdot}55$. Then find $P\{65 \le X \le 75\}$ 5

(d) Let $\underset{\sim}{X}_\alpha$ $(\alpha = 1, 2, \cdots, N)$ be a random sample from $N_P\left(\underset{\sim}{\mu}, \Sigma\right)$ and let

$$\underset{\sim}{\overline{X}} = \frac{1}{N}\sum_{\alpha=1}^{N} \underset{\sim}{X}_\alpha$$ be the sample mean vector.

Then prove that $\overline{X}$ is distributed as $N_P\left(\underset{\sim}{\mu}, \frac{\Sigma}{N}\right)$.

(e) (i) Let $\underset{\sim}{X}_\alpha^{(1)}(\alpha = 1, 2, \cdots N_1)$ be a random sample from $N_P\left(\underset{\sim}{\mu}^{(1)}, \Sigma\right)$ and let $\underset{\sim}{X}_\alpha^{(2)}(\alpha = 1, 2, \cdots N_2)$ be another random sample from $N_P\left(\underset{\sim}{\mu}^{(2)}, \Sigma\right)$ where the common dispersion matrix Σ is unknown. Discuss the procedure to test the hypothesis $H_0 : \underset{\sim}{\mu}^{(1)} = \underset{\sim}{\mu}^{(2)}$ using Hotelling's T^2 statistic. 5

(ii) In what way the ordinary sign test can be performed for paired samples? Explain. 5

(f) (i) State *any two* properties of multivariate normal distribution. 2

(ii) Derive the bivariate normal density as a particular case of multivariate normal distribution. 8

(g) (i) Let $\underset{\sim}{X} \sim N_3\left(\underset{\sim}{\mu}, \Sigma\right)$. Find the distribution of 5

$$\begin{pmatrix} X_1 - X_2 \\ X_2 - X_3 \end{pmatrix}$$

(ii) Derive the formula for Multiple correlation coefficient for a trivariate distribution. 5

(h) (i) Explain the distribution free method. 3

(ii) Derive the moment generating function of a bivariate normal distribution with usual parameters. 7
