Total number of printed pages-8

3 (Sem-6/CBCS) STA HC 2

2022

## STATISTICS

(Honours)

Paper : STA-HC-6026

(Multivariate Analysis and Nonparametric Analysis)

Full Marks : 60

Time : Three hours

## The figures in the margin indicate full marks for the questions.

- 1. Answer **any seven** of the following questions as directed : 1×7=7
- (a) The moment generating function of bivariate normal distribution with parameters  $(0, 0, \sigma_1^2, \sigma_2^2, \rho)$  is \_\_\_\_\_.

(Fill in the blank)

Contd.



(b) Let  $X \sim N_P(\mu, \Sigma)$ . Then the

characteristic of X is given by

(i) 
$$e^{i\underline{t},\underline{\mu}+\frac{1}{2}\underline{t}'\Sigma\underline{t}}$$
  
(ii)  $e^{i\underline{t}',\underline{\mu}-\frac{1}{2}\underline{t}'\Sigma\underline{t}}$   
(iii)  $e^{i\underline{t}',\underline{\mu}+\frac{1}{2}\underline{t}'\Sigma\underline{t}}$ 

(iv) None of the above

(Choose the correct option)

- Ordinary sign test considers the (c) difference of observed values from the hypothetical median value in terms of:
  - (i) signs only a short that
  - (ii) magnitudes only
  - (iii) sign and magnitude both
  - (iv) None of the above betoenib as (Choose the correct option)
- (d) What is dispersion matrix in Multivariate data analysis?
- (e) Let  $(X, Y) \sim \text{BVND}(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$ . Then state the conditional pdf of Y given X = x.

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method? (g) Define Multiple correlation coefficient. (h) Let  $X \sim N_3(\mu, \Sigma)$ . Given that sonsineverse (1 2 3)  $\Sigma = 2 3 0$ 3 0 4 Are  $X_2$  and  $X_3$  independent? The marginal distribution of a Bivariate (i) normal distribution follows univariate normal distribution. (State True or False)

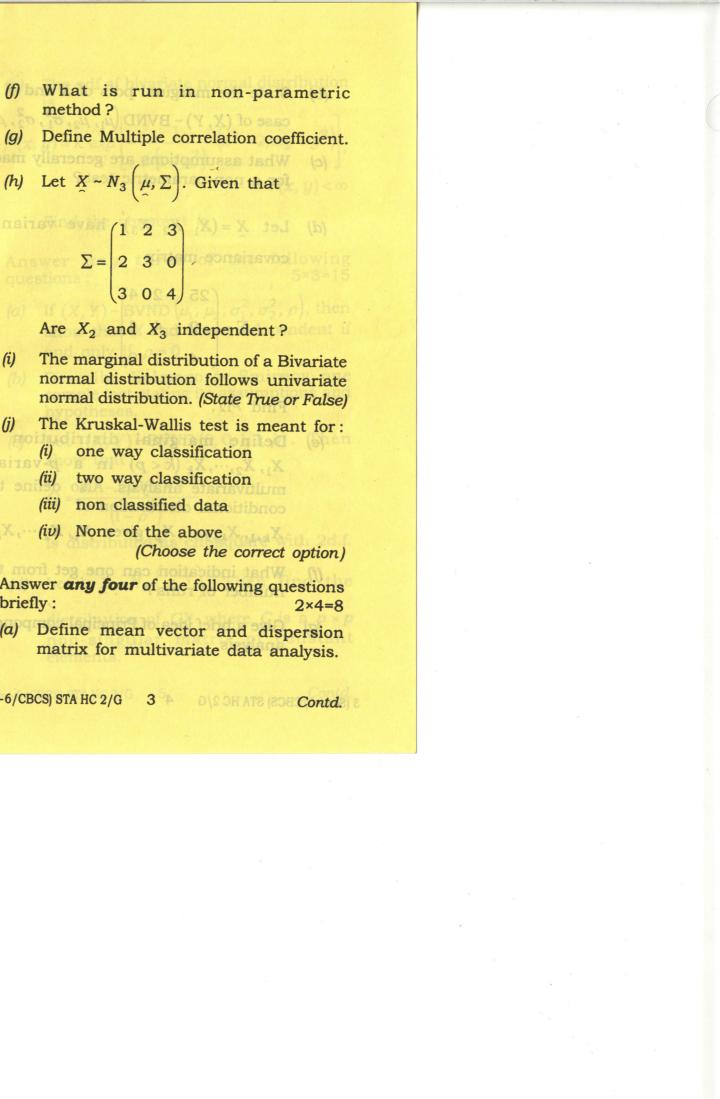
- The Kruskal-Wallis test is meant for: (i)
  - (i) one way classification
  - (ii) two way classification
  - (iii) non classified data

  - (iv) None of the above

(Choose the correct option)

- 2. Answer any four of the following questions briefly : 2×4=8
  - (a) Define mean vector and dispersion matrix for multivariate data analysis.

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- (b) State the marginal pdfs of X and Y in case of  $(X, Y) \sim \text{BVND}(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$ .
- What assumptions are generally made (c) for a non-parametric test?
- (d) Let  $X = (X_1 \ X_2 \ X_3)'$  have variance

covariance matrix

(25 -2 4)  $\sum = |-2| 4 1$ 1 9 Aut on of a Bivariate

Find  $P_{12}$ .

(e) Define marginal distribution of  $X_1, X_2, \dots, X_k \ (k < p)$  in a p-variate multivariate analysis. Also define the conditional distribution of

 $X_{k+1}, X_{k+2}, \dots, X_p$  given  $X_1, X_2, \dots, X_k$ .

- What indication can one get from the *(f)* number of runs?
- (g) Give a brief idea of Principal component analysis. terrevitium tot xintem

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(h) The pdf of bivariate normal distribution is

$$f(x, y) = k \exp \left[ -\frac{1}{2(1-\rho^2)} \left( x^2 - 2\rho xy + y^2 \right) -\infty < (x, y) < -\infty < (x, y) <$$

Find the constant k.

- 3. Answer any three of the following 5×3=15 questions :
  - (a) If  $(X, Y) \sim \text{BVND}(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$ , then show that X and Y are independent if and only if  $\rho = 0$ .
  - (b) Describe Kolmogorov-Smirnov one sample test stating its assumptions and
  - (c) Let  $(X, Y) \sim \text{BVND}(0, 0, 1, 1, \rho)$ . Then show that

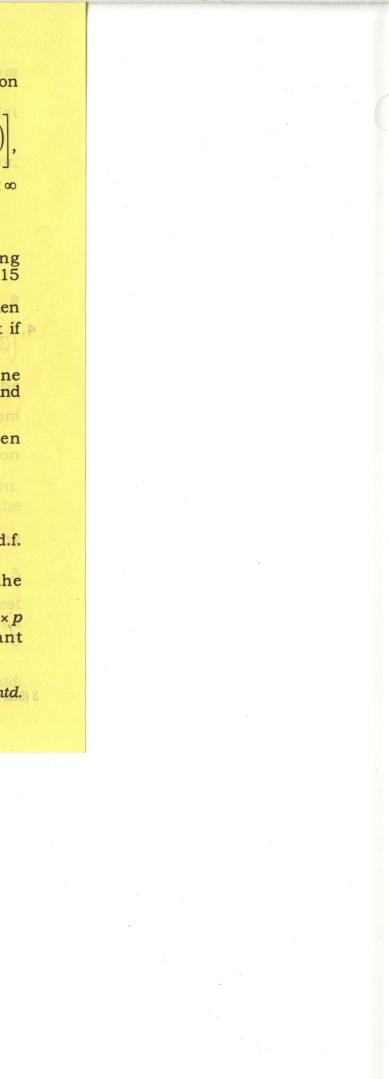
 $Q = \frac{X^2 - 2\rho XY + Y^2}{(1 + 1)^2}$ 

is distributed as chi-square with 2d.f.

(d) Let  $X \sim N_P(\mu, \Sigma)$ . Then find the distribution of CX where C is a  $p \times p$ 

non-singular matrix of constant elements.

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- Write an explanatory note on test of (e) randomness.
- With usual notations, prove that *(f)*

$$r_{12\cdot3} = \frac{r_{12} - r_{13}r_{23}}{\sqrt{\left(1 - r_{13}^2\right)\left(1 - r_{23}^2\right)}}$$

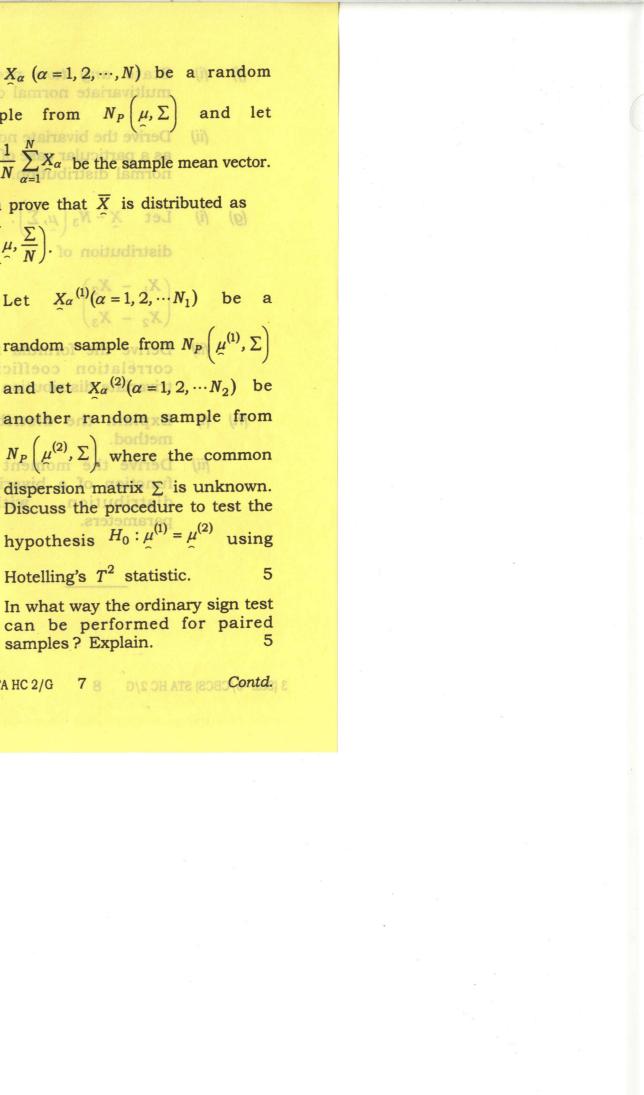
- Examine if Hotelling's  $T^2$  is invariant (g)under changes in the units of measurement.
- (h) Describe one sample sign test for testing the null hypothesis that the population median is a given value.
- 4. Answer any three questions from the 0 = 0 11 vino bo10×3=30 following :
- (a) (i) State any two applications of bas another multivariate analysis. 2
  - (*ii*) Let  $(X, Y) \sim \text{BVND}(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$ . Find the conditional distributions of X/Y=y and Y/X=x.
  - (b) Derive the probability density function of *p*-variate normal distribution.
- (c) (i) Describe the Wilcoxon Mann-Mhitney U test. dintalb al 5
- (ii) Let  $(X, Y) \sim BVND$  with parameters  $\mu_x = 60$ ,  $\mu_y = 75$ ,  $\sigma_x = 5$ ,  $\sigma_y = 12$  and  $\rho = 0.55$ . constant. Then find  $P\{65 \le X \le 75\}$ 5

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(d) Let  $X_{\alpha}$  ( $\alpha = 1, 2, \dots, N$ ) be a random and let sample from  $N_P \mid \mu, \Sigma \mid$ ormal density  $\overline{X} = \frac{1}{N} \sum_{\alpha=1}^{N} X_{\alpha}$  be the sample mean vector. Then prove that  $\overline{X}$  is distributed as  $N_P\left(\mu, \frac{\Sigma}{N}\right)$ . To noisedisteib (i) Let  $X_{\alpha}^{(1)}(\alpha = 1, 2, \dots N_1)$  be a (e) signification random sample from  $N_P\left(\mu^{(1)}, \Sigma\right)$ and let  $X_{\alpha}^{(2)}(\alpha = 1, 2, \dots N_2)$  be another random sample from  $N_P\left(\mu^{(2)}, \Sigma\right)$  where the common dispersion matrix  $\Sigma$  is unknown. Discuss the procedure to test the hypothesis  $H_0: \mu^{(1)} = \mu^{(2)}$  using Hotelling's  $T^2$  statistic. In what way the ordinary sign test (ii)

samples? Explain.

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a <b>() o m</b> nd let () vector.	(ii)	State any two properties of multivariate normal distribution. 2 Derive the bivariate normal density as a particular case of multivariate normal distribution. 8
(g)		Let $X \sim N_3\left(\mu, \Sigma\right)$ . Find the distribution of 5
(h) be a	Desi testi pour	$\begin{pmatrix} X_1 - X_2 \\ X_2 - X_3 \end{pmatrix}$
$\begin{pmatrix} a_{0} & A \\ b_{1} & a_{1} \\ \end{pmatrix}$ $\begin{pmatrix} a_{1} & b_{2} \\ a_{2} \end{pmatrix}$ be		Derive the formula for Multiple correlation coefficient for a trivariate distribution. 5
		Explain the distribution free method. 3
known. known. est the using	lhu e	Derive the moment generating function of a bivariate normal distribution with usual parameters. 7
	ary si for	$\frac{1}{6} \frac{1}{6} \frac{1}$
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