Total number of printed pages–11

3 (Sem–2/CBCS) CSC HC 2

2022

COMPUTER SCIENCE

(Honours)

Paper : CSC–HC–2026

(Discrete Structures)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following as directed : ***(any ten)*** 1×10=10

   (a) What do you mean by 'cut vertex' ?

   (b) Define recursive of a function.

   (c) What are predicates ?

   (d) Define a partially-ordered relation.

   (e) Every connected graph has maximum two spanning tree. *(State true **or** false)*

Contd.

(f) Every tree with two or more vertices is ________ chromatic. *(Fill in the blank)*

(g) Define Pigeonhole principle.

(h) Define a Binary tree.

(i) What is an antisymmetric relation ?

(j) Define the Big-O notation.

(k) Translate the following statement into mathematical logic :
"Some real numbers are rational"

(l) What is countably infinite set ?

(m) How many vertices are there in a tree with 20 edges ?

(n) Explain what it means for a function to be O (1).

(o) Explain what it means for a function to be Ω (1).

2. Answer ***any five*** of the following : 2×5=10

(a) In how many different ways, the letters of the word "GUWAHATI" can be arranged in a row if

(i) the two 'A's are together ?

(ii) the two 'A's are not together ?

(b) Define minimal spanning tree.

(c) What is the 'nullity' and 'rank' of a complete graph of *n*-vertices ?

(d) Show that $x^2+4x+17$ is $O\left(x^3\right)$ but that x^3 is not $O\left(x^2+4x+17\right)$.

(e) Define the Recurrence tree. How does the tree method help Recurrence relations ?

(f) Define the Quantifier in a logic.

(g) Derive the Recurrence relation for the following positive integers :

3, 6, 12, 24, 48, ...

(h) Show that the following logical expression are equivalent

$$P \vee \sim Q \equiv [(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)]$$

3. Answer ***any four*** of the following : 5×4=20

(a) Use the principle of mathematical induction to verify that

$$1+3+3^2+.....+3^{n-1}=\frac{3^n-1}{2}$$

(b) Define equivalence relation. If relation

$$R=\{(1,1),(2,3),(3,2),(2,2),(1,3),(3,1),(3,3)\}$$

on set $A=\{1,2,3\}$, determine whether R is a equivalence relation or not ?

(c) What are isomorphism of two graphs ? Show that the two graphs in *fig. 1* is not isomorphic.

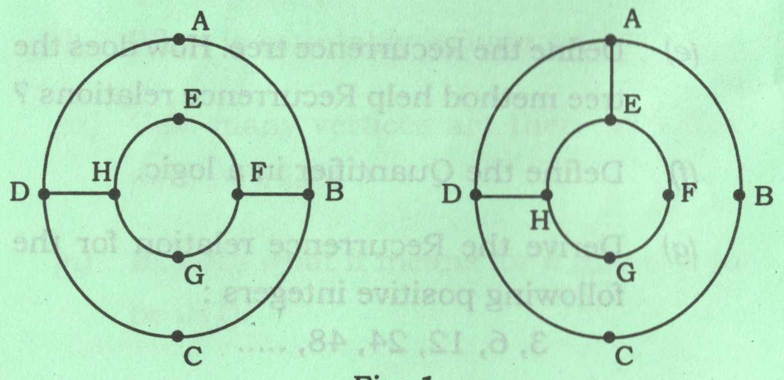


Fig. 1

(d) Solve the 1st order linear recurrence for $a_{n+1}=3a_n+4$, $a_1=2$.

(e) Determine whether the following argument is valid or not

(i) $p \rightarrow q$

$\underline{\sim p}$

$\sim q$

(ii) $p \vee q$

$\underline{\sim p \vee r}$

$q \vee r$

(f) Let k be a positive integer, show that $1^k+2^k+.....n^k$ is $O\left(n^{k+1}\right)$

(g) Define the following terms in a graph with diagrams :

Path, Walk, Cycle, Adjacency matrix, degree of a vertex.

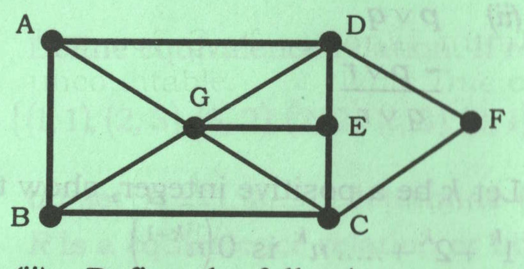
(h) Determine whether each of the following relations are reflexive, symmetric and transitive.

(i) $R=\{(x,y):3x-y=0\}$ in the set $A=\{1,2,3.....12,13\}$

(ii) $R=\{(x,y):y$ is divisible by $x\}$ in the set $A=\{1,2,3,4,5,6\}$

4. Answer ***any four*** of the following : 10×4=40

(a) (i) What is Hamiltonian circuit of a connected graph ? Find the Hamiltonian circuit using Backtracking approach for the given graph : 5



(ii) Define the following terms with an example for each planar graph : Incidence matrix, complement of a graph, pendant vertex and Euler graph. 5

(b) (i) When is a statement formula said to be tautology ?

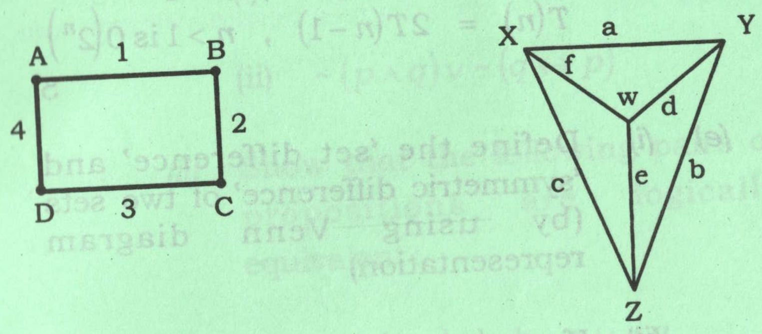
Show that $[(p \rightarrow q) \wedge p] \rightarrow q$ is a tautology. 5

(ii) What do you mean by normal form in logic ? Express $p \rightarrow (q \wedge r)$ in

(i) Disjunctive normal form (dnf)

(ii) Conjunctive normal form (cnf) 5

(c) (i) Find the adjacency matrix for the following two graph and determine that they are isomorphic : 6



(ii) Consider the following :

p : x is even

q : x is divisible by 4.

Write the following statements in logical form :

(a) x is even or x is divisible by 4

(b) x is even iff it is divisible by 4

(c) x is neither even or divisible by 4

(d) if x is even, then it is divisible by 4

4

(d) (i) Give a big-O estimate of the product of the first n-odd positive integer. 5

(ii) Show that for the recurrence equation,

$$T(n) = 1 \quad , \quad n=1$$
$$T(n) = 2T(n-1) \quad , \quad n>1 \text{ is } O(2^n)$$

5

(e) (i) Define the 'set difference' and 'symmetric difference' of two sets (by using Venn diagram representation)

(ii) If $A=\{1,2,3\}$, $B=\{5,6\}$, $C=\{2,3\}$, then find

(i) $(C\times B)-(A\times B)$

(ii) $A\oplus B\oplus C$

4+6=10

(f) (i) What is the difference between permutation and combination ?

(ii) A student buys two apples, four oranges and three mangoes from a person who had five apples, seven oranges and six mangoes. How many choices does the student have ?

4+6=10

(g) (i) Construct the truth tables for the following : 5

(i) $\sim p\vee q\rightarrow \sim q$

(ii) $\sim(p\wedge q)\vee \sim(q\leftrightarrow p)$

(ii) Show that the following pairs of propositions are logically equivalent : 5

(i) $\sim(p\wedge q)$ and $\sim p\vee \sim q$

(ii) $p\vee(p\wedge q)$ and q

(h) (i) Show that

$$1^2+2^2+3^2+\ldots\ldots+n^2=\frac{n(n+1)(2n+1)}{6}, \; n\geq 1$$

by mathematical induction. 5

(ii) Find the first four terms of each of the following recurrence relations :

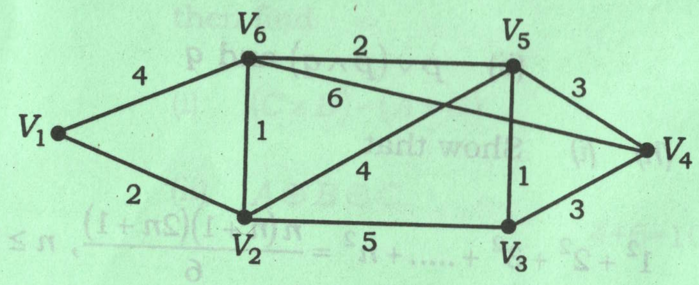
(i) $a_k=2a_{k-1}+k$, *for all integers* $k\geq 2$, $a_1=1$

(ii) $a_k = k(a_{k-1})^2$ *for all integers* $k \geq 1$, $a_0 = 1$

5

(i) (i) Prove that a tree with n-vertices has $(n-1)$ edges.

(ii) From the following weighted graph, find the minimum distance between vertex V_1 and V_4.



5+5=10

(j) (i) State and prove the principle of inclusion and exclusion for two sets A and B.

(ii) Among a group of students, 30 study physics, 35 study chemistry and 20 study maths. If 6 of these students study physics and chemistry, 8 students study chemistry and mathematics, 5 study physics and mathematics and 3 study physics, chemistry and mathematics. Find the number of students.

5+5=10

700