3 (Sem–6/CBCS) STA HE 3

2022

STATISTICS

(Honours Elective)

Paper : STA–HE–6036

(Actuarial Statistics)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Choose the correct options of the followings: ***(any seven)*** 1×7=7

(a) Range set of possible values of curtate future lifetime of (x) is

(i) $(-\infty, \infty)$

(ii) $(0, \infty)$

(iii) $\{0, 1, 2, \ldots\}$

(iv) $\{x, x+1, \ldots\}$

Contd.

(b) The present value of annuity certain immediate at the rate of 1 unit per annum for n years is given by

(i) $\frac{1-v^n}{d}$

(ii) $\frac{1-v^n}{\delta}$

(iii) $\frac{1-v^n}{i}$

(iv) $\frac{1+v^n}{d}$

(c) The relationship between T_x and L_x is

(i) $T_x = L_x + L_{x+1}$

(ii) $T_x = L_x - L_{x+1}$

(iii) $T_x = L_x + T_{x+1}$

(iv) $T_x = L_x - T_{x+1}$

(d) If $s(x)$ is survival function of X, then $s(0)$ is

(i) 0

(ii) 1

(iii) ∞

(iv) $\frac{1}{2}$

(e) The force of mortality μ_x is defined as the derivative w.r.t. x of

(i) $\log S(x)$

(ii) $-\log S(x)$

(iii) $\log F(x)$

(iv) $-\log F(x)$

(f) If w is limiting age, then

(i) $dw - 1 = 0$

(ii) $s(w) = 0$

(iii) $s(w) = 1$

(iv) $s(w) > s(0)$

(g) In equivalence principle, premium P is found such that

(i) $E(z) = PE(Y)$

(ii) $E(z) = E(Y)/P$

(iii) $E(z) = E(Y)$

(iv) $E(z) = E(P^2 Y)$

(h) The co-efficient of risk aversion is defined as

(i) $r(x) = -u''(x)/u'(x)$

(ii) $r(x) = -u''(x)$

(iii) $r(x) = u''(x)/u'(x)$

(iv) $r(x) = 1/u'(x)$

(i) If there is a maximum claim amount for the risk, say x_m, then

(i) the premium = x_m

(ii) the premium $\leq x_m$

(iii) the premium $\geq x_m$

(iv) $x_m = 0$

(j) Decision making using a utility function is based on

(i) the expected utility criterion

(ii) the utility function

(iii) the survival function

(iv) None of the above

2. Answer the following questions : **(any four)** 2×4=8

(a) Explain the term 'loss function'.

(b) Define curtate future lifetime random variable $K(x)$.

(c) Explain the concept of utility function.

(d) What are the two sources of uncertainty for the insurer ?

(e) Name any two methods by using which one can find the distribution of sum of random variables.

(f) State any two properties of survival function.

(g) Explain the concept of pure premium principle

(h) What is premium loading factor ? Explain.

3. Answer the following questions : **(any three)** 5×3=15

(a) What is reinsurance arrangement ? Explain proportional reinsurance arrangement.

(b) A random variable X has a logarithmic distribution with parameter θ, where $0 < \theta < 1$, if its probability function is

$$Pr(X = x) = \frac{-1}{log(1-\theta)} \cdot \frac{\theta^x}{x} \text{ for } x = 1, 2, 3, \ldots$$

Show that

$$M_x(t) = \frac{log\left(1-\theta e^t\right)}{log(1-\theta)} \text{ for } t < -log\,\theta.$$

Hence, or otherwise, find the mean and variance.

(c) For a utility function, $u(x) = -exp(-0.002x)$, two investments give net gains

$$X_1 \sim N\left(10^4, 500^2\right) \text{ and } X_2 \sim N\left(1.1x10^4, 2000^2\right)$$

Which of these investments does the investor prefer ?

(d) An insurer, whose current wealth is W, uses the utility function

$$u(x) = x - \frac{x^2}{2\beta}$$

where $x < \beta$, for decision making purposes. Show that the insurer is risk averse, and that the insurer's risk aversion co-efficient, $r(x)$, is an increasing function of x.

(e) Explain the expected value principle of examples of premium principles.

(f) A premium principle is said to be sub-additive if for two risks X_1 and X_2 (which may be dependent), $\Pi_{x_1+x_2} \le \Pi_{x_1} + \Pi_{x_2}$. Under what conditions is the variance principle sub-additive ?

(g) Derive the expression for $\mu(x)$ (force of mortality) in terms of the survival function $S(x)$.

(h) Explain individual risk model.

4. Answer the following questions : ***(any three)*** 10×3=30

(a) Explain the method of direct convolution of distribution to find the distribution of sums of random variables.

(b) With an example explain the exponential utility function.

(c) An insurer has offered an individual insurance cover against a random loss, X, where X has a mixed distribution with distribution function F given by

$$F(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1 - 0.2e^{-0.01x} & \text{for } x \ge 0 \end{cases}$$

The insurance cover includes a policy excess of 20. Calculate the minimum premium that the insurer would accept if the insurer bases decisions on the utility function $u(x) = -e^{-0.005x}$.

(d) Explain the properties of premium principles.

(e) Explain the principle of zero utility of premium of principles.

(f) The mean value principle states that the premium Π_x , for a risk X is given by

$$\Pi_x = v^{-1}(E[v(X)])$$

where v is a function such that $v'(x) > 0$ and $v''(x) \ge 0$ for $x > 0$.

(i) Calculate Π_x when $v(x) = x^2$ and $X \sim \gamma(2, 2)$.

(ii) Construct a counter example to show that this principle is not consistent.

(g) Explain the columns of the life table.

(h) Let X be the age at death random variable with

$$\mu_x = \frac{1}{2(100-x)} \text{ for } 0 \le x < 100 .$$

(i) Find the survival function of X.

(ii) Find $f_{36}(t)$, the density function of future lifetime of (36).

(iii) Compute ${}_{20}P_{36}$, the probability that life 36 will survive to reach age 56.

(iv) Compute the average future lifetime of 36.
