Total No. of printed pages = 10

3 (Sem 3) PHY M1

2015

PHYSICS

(Major)

Paper : 3.1

(Mathematical Methods-III and Electrostatics)

Full Marks – 60

Time – Three hours

The figures in the margin indicate full marks for the questions.

GROUP–A

(Mathematical Physics)

Marks : 25

1. Answer the following questions : 1×3=3

   (a) Define rank of a matrix.

   (b) When a symmetric matrix is said to be Hermitian ?

   (c) Find adj A if $A = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

[Turn over

2. Check whether the total angular momentum matrix for an electron given by

$$J^2 = (3\hbar^2/4)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \text{ is}$$

Hermitian and unitary. 2

3. Answer any *two* of the following questions : $2\times5=10$

(a) (i) If $A^2 + A - I = O$ find A^{-1}. 1

(ii) Show that the inverse of a matrix is unique. 2

Or

If A, B are two square, symmetric matrices of dimensions $n \times n$ find the condition when the product AB is symmetric.

(iii) Find the rank of the matrix 2

$$\begin{pmatrix} 3 & -1 & 2 \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{pmatrix}$$

(b) (i) Express the following quadratic form as product of matrices

$$ax^2 + by^2 + 2hxy$$ 2

(ii) What is an idempotent matrix ? 1

(iii) Find the value of b if the matrix

$$\begin{pmatrix} 0 & 1 & b \\ -1 & 0 & 4a \\ 2a & 2b & 0 \end{pmatrix}$$

is skew-symmetric. 2

(c) (i) Show how centrifugal force and coriolis force appear in the description of motion of a particle in the frame fixed with respect to rotating earth. 4

(ii) "Sum of finite rotations performed on a rigid body depends on the order of rotations."

What operation on two transformation matrices A, B are linked with the statement ? 1

4. Answer either [(a) and (b)] or [(c) and (d)].

(a) (i) Given the matrices

$$A = \begin{pmatrix} 0 & 2 & 3 \\ 3 & -1 & 1 \\ 4 & 2 & 1 \end{pmatrix}, \quad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad C = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

and the equation $AX = C$.

Solve for x, y, z by matrix method. 3

(ii) Test whether the matrix

$$\begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

is orthogonal or not. 2

(b) (i) Consider the following transformation in three dimensions 1+2=3

$x' = x\cos\theta + y\sin\theta$

$y' = -x\sin\theta + y\cos\theta$

$z' = z$

Write down the transformation matrix A.

Show that $A(\theta_1 + \theta_2) = A(\theta_1) + A(\theta_2)$.

(ii) Find the inverse of the matrix 2

$$\begin{pmatrix} 2 & -3 \\ 4 & 6 \end{pmatrix}$$

(c) (i) Test whether the following matrices are diagonalizable : 1½+1½=3

$$A = \begin{pmatrix} 1 & 1+i \\ 1-i & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$$

(ii) State and explain Cayley - Hamilton theorm. 2

(d) (i) Calculate the eigen values of M, M^{-1} where $M = \begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{pmatrix}$ 2+1=3

(ii) If $A = \begin{pmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{pmatrix}$ and I is a unit matrix, show that 2

$$I + A = (I - A)\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$$

GROUP–B

(Electrostatics)

Marks : 35

5. Choose the correct option : 1×4=4

(a) The induced surface charge q' is related to charge q as (where K = dielectric constant)

(i) $q' = q/k$ (ii) $q' = q$

(iii) $q' = q\left(1 - \frac{1}{K}\right)$ (iv) $q' = q(1-K)$

(b) The electric potential due to a quadrupole at distance far off from its centre varies as

(i) $1/r$ (ii) $1/r^2$

(iii) $1/r^3$ (iv) $1/r^4$

(c) Electric field at a point close to the surface of a charged conductor having charge density σ is

(i) $\sigma/4\epsilon_0$ (ii) σ/ϵ_0

(iii) $\sigma/2\epsilon_0$ (iv) $\sigma/3\epsilon_0$

(d) Unit of electric potential in terms of base units of SI is

(i) $Kg\ m^2S^{-1}$ (ii) $Kg\ m^2S^{-1}A^{-1}$

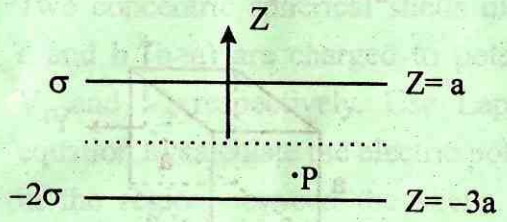
(iii) $Kg\ m^2S^{-2}$ (iv) $Kg\ m^2S^{-3}A^{-1}$

6. Answer the following questions : $3\times2=6$

(a) Two charges $q_1=2\mu C$ and $q_2=3\mu C$ are placed at (0, 0, 4) and (0, 0, –4) respectively. Find the locus of points where potential is zero. 3

Or

What is the net electric field at the point P due to the two infinitely long charged sheets as shown in the figure.



(b) (i) Can the potential have a maximum or minimum in free space ? Justify. 1½

(ii) Show mathematically that $\nabla\phi$ is perpendicular to the equipotential surface $\phi(x, y, z) =$ constant. 1½

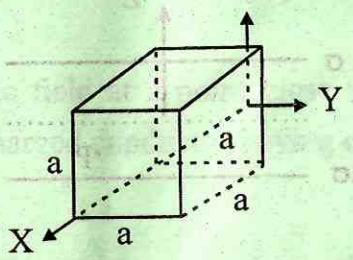
7. Answer either (a) or (b)

(a) (i) Compute the curl and divergence of electric field given by

$$\vec{E}(x, y, z) = (x^2 + z^2 + 5)\hat{x} + (y^2 + x^2 - 9z)\hat{y} + (z^2 + y^2)\hat{z}.$$

Is the field conservative ? 1+1+1=3

(ii) For the electric field $\vec{E} = (Kx^{1/2})\hat{x}$ where K is a constant, compute the electric flux through the face of the cube region depicted in the figure. 2



(b) (i) Using Gauss's law find an expression for the mechanical force per unit area on the surface of a charged conductor. 2½

(ii) Derive an expression for the equilibrium of an electrified soap bubble. 2½

8. Answer any *two* questions :

(a) (i) Four charges $q_1 = q_2 = -q_3 = -q_4 = q$ are arranged at points (0, a), (0, –a), (a, 0) and (–a, 0) in x–y plane. What is the electrostatic energy needed to compose such an arrangement, bringing the charges in from infinity ? Also find the electrostatic potential in x–y plane. 2+3=5

(ii) Show that an arbitrary distribution of charges is equivalent to combination of monopole, dipole, quadrupole etc. 5

(b) (i) Two concentric spherical shells of radii a and b (b>a) are charged to potentials V_1 and V_2 respectively. Use Laplace's equation to calculate the electric potential in the region between the shells. 5

(ii) Use Poisson's equation to find the charge density in a region where the potential function is

$V = a + 4b\ (x^2+y^2) - c \log\ (x^2+y^2)^{3/2}$. 5

(c) (i) What is meant by dielectric polarisation? Show how $\vec{E}, \vec{D}, \vec{P}$ are related for an isotropic dielectric medium. Is water molecule a polar molecule ? If so, why ? 1+3+2=6

(ii) Using Clausius-Mosotti relation show how can the atomic radius be determined from dielectic constant. 4

(d) (i) Determine the electric potential at a point on the axis of a charged disc of radius a and surface charge density σ. Show that the disc may be regarded as a point charge for far off points. 3+2=5

(ii) Two equal point charges q are placed at equal distance b from the centre of a grounded conducting sphere of radius a where $b<a$. Calculate the force acting on each of the charges. 5